Statistical Assessment of Model Fit for Synthetic Aperture Radar Data

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Motivation

- Many recognition approaches are based on statistical models
- Models describe conditional distributions of observations
- Accuracy of models is key for good performance
- Much prior work on marginal (spatial) distributions for radar detection
- Conditional assessment involves large numbers of small samples

Outline

1. Statistical hypothesis testing
2. Conditional models for SAR imagery
3. Unknown parameters
4. Goodness-of-fit
5. Conclusions
Hypothesis Testing

For each model assumption evaluate

\( H_0: \) model valid \hspace{1em} vs. \hspace{1em} \( H_A: \) model invalid

- Find test statistic \( D(r_1, r_2, \ldots, r_N) \), with known distribution under \( H_0 \)
- P-value is probability of a more extreme observation if \( H_0 \) were true:
  \[ P = 1 - F_D(d) = \Pr[D > d \mid H_0] \]
- Small values of \( P \) evidence for rejecting \( H_0 \).
- P-value uniformly distributed when \( H_0 \) true

P-value distribution for test of symmetry
# SAR Dataset

Data for analysis taken from 10 targets in the MSTAR dataset

<table>
<thead>
<tr>
<th>Target</th>
<th>Vehicles</th>
<th>Images (15° Depression)</th>
<th>Images (17° Depression)</th>
</tr>
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<tbody>
<tr>
<td>2S1</td>
<td>b01</td>
<td>274</td>
<td>299</td>
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<td>BRDM-2</td>
<td>E-71</td>
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<td>ZSU 23 4</td>
<td>d08</td>
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</tbody>
</table>

- 6862 images at 2 depression angles and hundreds of azimuth angles
- Pixels on target from 20 x 20 square in image center
- Pixels on clutter from 20 x 20 square in top-left corner
Complex Gaussian Model

Each pixel is independent, complex-valued with mean zero and variance a function of target class, pose, and pixel location

\[
p(r|a, \theta) = \frac{1}{\pi \sigma^2(a, \theta)} e^{-\frac{|r|^2}{\sigma^2(a, \theta)}}
\]

Can relax the zero-mean assumption, allowing mean to be a function of target class, pose, and pixel location

\[
p(r|a, \theta) = \frac{1}{\pi \sigma^2(a, \theta)} e^{-\frac{|r-\mu(a, \theta)|^2}{\sigma^2(a, \theta)}}
\]
Normal Transformation Models

Transformation of pixel magnitude yields Gaussian distribution

Lognormal: \[ x = \log |r| \text{ is Gaussian} \]

Quarter power normal: \[ x = |r|^{1/2} \text{ is Gaussian} \]

Transformed values are independent with mean and variance that are functions of target class, pose, and pixel location

Homoscedastic assumption: constant variance

\[
p(x|a, \theta) = \frac{1}{\sqrt{2\pi \sigma^2 (a, \theta)}} e^{-\frac{(x-\mu(a, \theta))^2}{2\sigma^2 (a, \theta)}}
\]
Large Numbers of Small Samples

- Distribution parameters $\mu(a, \theta)$ and $\sigma^2(a, \theta)$ are unknown
- Assess model validity over $\Delta \theta = 12^\circ \Rightarrow 10$-20 samples per interval
- Kolmogorov method requires known parameters
- $\chi^2$ method can use estimates but needs large samples

Convert independent Gaussian variates with unknown parameters to dependent T variates with known DOF

Test for T distribution

Given independent $Z_1, Z_2, \ldots Z_N$ Gaussian with unknown $\mu, \sigma^2$

\[
(Z_k - \bar{Z}) \sqrt{\frac{N-1}{N} S^2(k)}
\]

is T distributed with $N - 1$ DOF, where

\[
S^2(k) = \frac{1}{N-1} \sum_{j \neq k} (Z_j - \bar{Z}(k))^2
\]

and

\[
\bar{Z}(k) = \frac{1}{N-1} \sum_{j \neq k} Z_j
\]
D’Agostino-Pearson Method

Let $P_{G_1}(g_1)$ and $P_{G_2}(g_2)$ be the cumulative distribution functions for sample skewness and kurtosis of $N$ Gaussian random variables

$$g_1 = \frac{k_3}{k_2^{3/2}} \quad \text{and} \quad g_2 = \frac{k_4}{k_2^2}$$

where $k_i$ is the $i$th $k$-statistic

D’Agostino-Pearson statistic $D_{DP}$ given in terms of the inverse CDF of a standard normal variate, $\Phi^{-1}(z)$

$$D_{DP} = \left[ \Phi^{-1}(P_{G_1}(g_1)) \right]^2 + \left[ \Phi^{-1}(P_{G_2}(g_2)) \right]^2$$

$Pr[D_{DP} > d] = 1 - P_D(d)$ is the P-value of the test

Distributions $P_{G_1}, P_{G_2}$ and $P_D$ approximated from 1,000,000 samples
D’Agostino-Pearson Method

- Quarter-power normal a good match, especially for clutter
- Gaussian model results unaffected by zero-mean assumption
- Log-normal model not as accurate for clutter pixels

Pixel on Target

Pixel on Clutter
Pearson’s $\chi^2$ Method

Test for T distribution of converted data with binned observations

- Partition $\mathcal{R}$ into $K$ intervals
- Let $q_k$ be the probability of interval $k$ from the T distribution
- Let $N_k$ count the observations in interval $k$
- Expected number of observations is $Nq_k$ under $H_0$

Then, $D^2 = \sum_{k=1}^{K} \frac{(N_k - Nq_k)^2}{Nq_k}$ is asymptotically $\chi^2_{k-1}$ distributed as $N \rightarrow \infty$

$\Pr[D^2 > d] \approx 1 - P_{\chi}(d)$ is the P-value of the test
Pearson’s $\chi^2$ Method

- Quarter-power normal a good match for clutter
- Zero-mean assumption yields a more powerful test
- Lognormal a particularly poor fit for clutter
- No model accurately characterizes target pixels

![Graphs showing cumulative probability vs P-value for different models](image1)

- **Pixels on Target**
- **Pixels on Clutter**
Kolmogorov-Smirnov Method

Test for T distribution of converted data from empirical CDF

From observations $R_1, R_2, \ldots, R_N$ determine the empirical CDF

$$\hat{P}_R(r) = \frac{1}{N} \sum_{i=1}^{N} u(r - R_i)$$

where $u(\cdot)$ is the unit-step function

The Kolmogorov-Smirnov test statistic is the supremum

$$D_{KS} = \sup_{r} |\hat{P}_R(r) - P_R(r)|$$

$\Pr[D_{KS} > d]$ is the P-value of the test

Kolmogorov-Smirnov Method

- Gaussian and quarter-power normal a better fit for clutter than target
- Lognormal less accurate for ground clutter
Conclusions

• Method to directly assess models over large numbers of samples
• Ordering of curves is consistent across all tests conducted
  Quarter-power normal consistently a better fit
• Each model is not unreasonable for most samples on target
• Only lognormal is unreasonable for most samples on clutter
• Gaussian and quarter-power normal better fit for clutter
• Strong evidence that none of the models is completely accurate

Future Work

• Evaluation of other model claims in progress
• Examine generalizations of these models
• Incorporate other models for SAR imagery